Estimating Delay of Vehicles in Nearside Legs of the Signalized Intersections under Expectation Method in Under-Saturation Conditions for Isolated Intersection

Iraj Bargegol¹, Vahid Najafi Moghaddam Gilani²

¹Department of Civil Engineering, Faculty of Engineering University of Guilan, Rasht, Iran.
²Highway & Transportation Engineering, Faculty of Engineering University of Guilan, Rasht, Iran.
Correspondence: Iraj Bargegol (Bargegol@guilan.ac.ir)

Abstract: Signalized intersections are essential elements of road transport in cities, thus providing condition in which these key elements have optimal performance in different traffic conditions has always been a concern for traffic engineers. Normally, Average vehicle control delay (delay due to the presence of traffic lights) is used as performance indicators on signalized intersections that is estimated using a lot of equations, including the Webster, Green Shields and the equation described in the book “Highway Capacity Manual”. But in these equations no place is include for the distribution of vehicles on the routes leading to the signalized intersections. Ignoring this issue could lead to a false estimate of the average amount of delay on the signalized intersections because of the type of their entrance resulting in inefficient operation of the intersection. Using Expectation Method, this paper proposes a better estimate of the amount of delay on the vehicles in signalized intersections, depending on frequency of entrance to the intersection. In the end it will be found out that different frequency distribution of vehicles at nearside legs of intersection lead to different delay estimates of vehicles.

Keywords: Expectation method, the expected values, the delay of vehicles, entrance frequency distribution.

Introduction

Urban transport network is made up of two parts: roads and intersections. Considering the fact that intersections are considered as urban transport network nodes, they impose maximum delay to vehicles¹. Proper estimation of delay of vehicles in order to find a suitable timing on signalized intersections is one of the main ways to solve the problem of traffic nodes in cities.

The purpose of this study is to develop an analytical method to estimate the controlled delay of vehicles on nearside signalized intersection based on controlled delay equation mentioned in the book of “Highway Capacity Manual” for under saturation conditions. Therefore, this paper examines the control delay changes based on random variables affecting an isolated intersection, in under-saturated conditions, using Expectation method. Variables such as the amount of vehicles, the green time and saturation flow rate that affect the calculation of the average delay caused by traffic lights are random variables that obey their statistical distribution characteristics.

Three models were proposed by Webster, Mailer and Noel on estimating delay imposed on vehicles; later Hatkinson, Susain and Kranch compared these models analytically. The most famous equation to estimate the average delay control for vehicles is the equation mentioned in “Highway Capacity Manual”. The input parameters in the equation are: intersection geometry, traffic conditions and the timing of traffic lights. Considering the type of signalized intersection and traffic conditions governing it, the average traffic flow and average saturation flow rate of vehicle are sued to estimate the controlled delay average of vehicle using HCM equation and there is no place for the type of frequency distribution of vehicles in nearside legs of intersection. So in this article we have tried to offer a method to estimate a more accurate delay with respect to the frequency distribution of vehicles in nearside legs of intersection.

A review of previous studies

More than 40 years, many models have been proposed to calculate and estimate delays of vehicles on signalized intersections. One of the first models to estimate the delay was Vardrope model released in 1952. Vardrope assumed that vehicles reach the intersection steadily. In this model, 

$$d = \frac{(r - \frac{1}{2s})^2}{2c(1 - y)}$$

Vardrope found out that the term is a small amount compared to r term that can be ignored. Vardrope Estimation of delay equation is as follows.

Where $d$: average delay imposed on the vehicle (seconds), $r$: Effective red time duration (seconds), $S$: saturation flow rate to the desired nearside leg (vehicle per hour), $C$: Cycle length (seconds), $y$: flow ratio.

In the following years three other models were proposed by Webster, Mailer and Noel on estimating delay imposed on vehicles; later Hatkinson, Susain and Kranch compared these models analytically. Hatkinson corrected Webster's delay model by introducing $I$ variable. When the value of variable $I$ is equal to one, Hatkinson and Webster equation will be the same. Hatkynson equation is expressed as follows\[10]\:

$$d = 0.9 \left( \frac{C \left( 1 - \frac{g}{C} \right)^2}{2 \left( 1 - \frac{g}{C} \right) \left( \frac{y}{C} \right)} + \frac{I x^2}{2y(1-x)} \right)$$

Where $C$: Cycle length, $g$: Effective green time, $v$: the rate of flow of vehicles, $x$: saturation degree.

The analyses of Hatkinson showed that the delay obtained by Webster equation calculates relatively low delay when $I$ is greater than 1 and saturation degree is high. He also claimed that the improvement of Webster equation with the variable $I$ is a suitable method for estimating the random delay.

Based on empirical studies, Van presented a model for estimating the delay. In this model, by changing the average variance variable of the rate of input flow during each cycle($1_a$) into the average variance variable of the rate of output during each cycle($1_a$) in Hatkinson’s model presented the following equation\[11]\:

$$I_d = I_a \exp(-1.3F^{0.527})$$

$$F = \frac{Q_a}{(I_a \nu C)^{6.5}}$$

Where $Q_a$: The average queue at saturated level (vehicle)

Statement of the problem

In nearside legs of intersections the random entrance of the vehicles lead to different delays imposed on the vehicles. Previously, the calculation of delay on vehicles was calculated by various relations including Webster, Aksilic, HCM2000 and etc, as points and used for the average saturation values. But in this paper, using various parameters such as frequency distribution, specified average and variance values of vehicles at nearside legs of intersections it is possible to calculate changes at an acceptable level of confidence.

Determining the estimation methods of vehicles’ delay

The method presented in this article is an analytical method based on the equations governing the mathematics and statistics. Accordingly in order to estimate the delay of the vehicles at the nearside legs of intersections the Expectation Method is used.

The equation of the delays on the vehicles in “Highway Capacity Manual” has a lot of input variables to calculate the delay on the vehicles including the length of cycle, the saturation flow rate, capacity and etc. In this article assuming other variables as constant, the vehicles entering the intersection are supposed to be variable.

Each random variable has specific frequency distribution with specific average and variance that are the characteristics of each distribution.

Assuming that the average and variance of input capacity and distribution of vehicles entering the intersection have specific values, it is possible to achieve average and variance of saturation degree using the equation (4). Also based on the equation (4) the frequency distribution of saturation level of the vehicles is equal with the frequency distribution of the vehicle entering the intersection.

$$X = \frac{v}{c}$$

Where $X$: Saturation level of the nearside legs of intersection, $V$: the rate of the vehicles entering into the intersection and $c$: the capacity of the nearside legs of intersection.

Given that saturation flow rate, the green time of each phase and the length of the cycle are constant, based on the above assumptions, the capacity of the nearside legs of the intersection is obtained by equation (5).

$$c = S \times \frac{g_i}{c}$$

Where $c$: the capacity of each nearside legs of the intersection, $S$: Saturation flow rate, $g_i$: The green time of the desired phase and $C$: the cycle length of traffic light.

Therefore, according to equation (4) the average degree of saturation equals the average input rate of the vehicles entering the intersection; also according to equation (5) saturation variance equals the variance in the vehicles entering the nearside legs divided by the square capacity.

Knowing the type of saturation frequency and the expectation equation the expectation values for various exponents of degree of saturation ($X$) can be calculated based on the equation.

$$E[X^n] = M^{(n)}(t = 0)$$

Where $E [X^n]$: the expected values for the saturation level and $M (t)$ torque generator function.

Delay function equation of HCM vehicles is as equation (7) that after calculating the expected saturation, the amount of delay of vehicles is estimated by HCM delay equation through Taylor series expansion. Based on the assumptions in this article the only random variable in equation (7) is the degree of saturation.
Where C : The cycle length, g : the green time for the desired phase, X : Saturation and c: Capacity

For the vehicle at nearside leg it should be noted that due to considering random inputs of the vehicles at the nearside leg is the type of arrival of the vehicle to the intersection (AT) based on the third type i.e random inputs (AT3). That is why the correction coefficient of pack movement (PF) is considered as 1 and considering that the considered intersection is isolated I equals 1 and due to the scheduled traffic light K is equal to 0.5 and the time of analysis (T) is assumed to be equal to 15 minutes. As discussed before this equation due to the assumption made is solely dependent on the degree of saturation (X) random variable.

In order to estimate the delay of vehicles the Taylor expansion of HCM vehicle delay which is subject to saturation variable is used. Taylor series is expanded around the average value of saturation to obtain a suitable estimation of the delay of vehicles based on the type of vehicles entry into the intersection. Taylor series expansion function of delay (D), around the average saturation point (X₀) is presented in equation (8).

\[ D(X) = \sum_{n=0}^{j} \left( \frac{1}{n!} \frac{d^n D(X_0)}{dx^n} \right) (X - X_0)^n \]

Where D (X): delay estimation function obtained from delay function HCM and X₀: The amount of average saturation level

The value of j is estimated based on the fact the how much the estimated equation reflects the delay values obtained from HCM equation. But normally for the saturation levels less than 1 the values of 3 or 4 are suitable. But in this paper, Taylor series is expanded up to the fourth exponent (X) (i.e. j equals 5). After calculating the delay function of the vehicles using Taylor series expansion the expected values of the saturation flow rate that are estimated by expectation function and based on the type of the entrance of the vehicles to the intersection are placed in the delay function and the vehicles’ delay is obtained.

For example, for the cycle length of 45 seconds and the green time of 20 seconds and average saturation of 0.5 and 0.85 using the discussed method it is possible to estimate the delay function of the vehicles at the nearside leg versus the HCM delay function.

![Figure 1. Delay function of the vehicles for g = 20, C = 45 and average saturation degree 0.5](image1)

![Figure 2. Delay function of the vehicles for g = 20, C = 45 and average saturation degree 0.85](image2)

As the figures (1) and (2) indicate this method results in rational and close to the delay obtained from HCM delay functions in average lower saturations. By knowing the distribution frequency, the average value and variance of saturation (X) it is possible to calculate he range of saturation change at defined confidence level (-α 1), using equation (8) and then by having the delay estimation function in that range of saturation changes, the delay imposed on the vehicles is obtained.

\[ C.I = \mu \pm \sigma \times I \]

Where CI : Interval changes, μ: Average, σ: standard deviation, I : the number read based on the desired confidence level and saturation distribution from the statistical tables.

By obtaining range of saturation changes in a specified confidence level, it can be concluded that how the delay resulting from the described method presents the changes in the delay of vehicles at nearside legs.

In order to analyze the effect of various distribution of the vehicles at nearside legs of intersection on the delay estimation function, three different input distribution of normal, Poisson and uniform are considered for the vehicles. Also delay estimation function for three different input distributions were analyzed. For instance the diagram of three average saturation s of 0.5, 0.85 and 0.9
are presented to evaluate their effect on delay estimation function.

The results for the average saturation of .5, 0.85 and 0.9 and various delay distributions are as follows.

![Figure 3. Delay estimates under different input distribution C = 93 and g = 45 s and average saturation X=0.5](image)

![Figure 4. Delay estimates under different input distribution C = 93 and g = 45 s and average saturation X=0.85](image)

![Figure 5. Delay estimates under different input distribution C = 93 and g = 45 s and average saturation X=0.9](image)

After lots of analyses and with regard to Figure 3, the delay estimation function of vehicle properly predicts vehicles’ delay low degree of saturation(less than 0.8) but as the value of degree of saturation increases this function predicts values higher than the value obtained by HCM delay function. As previously stated and it is clear from the figures at low degree of saturation the delay estimation function predicts values close to values estimated by HCM function and as the value becomes greater the difference becomes wider.

At average saturation of less than 0.8 there is significant difference between different input types the amount of estimated delay but the delay caused by the entry of Poisson of the nearside legs vehicle into the estimated delay, it is closer than the HCM equation and then the normal and uniform distribution predict values closer than the values obtained by HCM. In the average high saturation levels (more than 0.8 and less than 0.9) the difference between the estimated values by HCM is higher than the delays estimated in low average saturations. In this average levels of saturations by increasing the saturation level the difference between the estimated delay and obtained delay by HCM is reduced as in low degrees of saturation the difference is increased and at higher degrees it is reduced. At this average saturation the least difference between the estimated and calculated values by HCM equation is achieved by uniform distribution and then it is obtained by Poisson and finally the normal.

At high average saturation (more than 0.9) the difference between the estimated values of delay resulted by various frequency distributions and the estimated value by HCM is much more than the estimated delay at average saturation lower than 0.8 and 0.9. In this average levels of saturations by increasing the saturation level the difference between the estimated delay and obtained delay by HCM is reduced as in low degrees of saturation the difference is increased and at higher degrees it is reduced. At this average saturation the least difference between the estimated and calculated values by HCM equation is achieved by uniform distribution and then it is obtained by Poisson and finally the normal.

**Conclusion**

The results can be summarized as follows:

Different input distribution of vehicles at nearside legs of the signalized intersections lead to different delays between the vehicles.

Different average degree of saturation is the result of various sizes of vehicles at nearside legs of intersection that lead to various delays for the vehicles.

Delays resulting from the presented method provide sensible results at low average saturation levels.

Given the type of the frequency distribution of the input vehicles at the nearside legs of the signalized intersections and obtaining the average and variance saturation using equation (4) and equation (9) it is possible to explain under certain confidence that is a range of saturation changes how much delay is imposed on the vehicles entering the nearside intersections.

**References**


